

# A Fixpoint Based Encoding for Bounded Model Checking

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**Abstract.** The *Bounded Model Checking* approach to the LTL model checking problem, based on an encoding to Boolean satisfiability, has seen a growth in popularity due to recent improvements in SAT technology. The currently available encodings have certain shortcomings, particularly in the size of the clause forms that it generates. We address this by making use of the established correspondence between temporal logic expressions and the fixpoints of predicate transformers as used in symbolic model checking. We demonstrate how an encoding based on fixpoints can result in improved performance in the SAT checker.

## 1 Introduction

Bounded Model Checking (BMC) [2] is an encoding to Boolean Satisfiability (SAT) of the LTL model checking problem. The encoding is achieved by placing a bound on the number of time steps of the model that are to be checked against the specification. The resulting Boolean formula contains variables representing the state variables of the model at each step along a path, together with constraints requiring the path to be contained within the model and to violate the specification. The result of the SAT checker is thus a path in the model which is a counterexample to the specification, or failure, which means that no such path exists within the bound.

The encoding of the LTL specification in BMC is defined recursively on the structure of the formula. While for simple specifications this is sufficient, more complex specifications such as bounded existence and response patterns [7] lead to an exponential blowup in the size of the resulting Boolean formula. Recent improvements to the encoding in NuSMV [4] have not removed this restriction.

The fixpoint characterisations of temporal operators [8] have been exploited in other model checking systems such as SMV [14]; we discuss an approach to their use in an encoding of LTL for BMC which produces more compact encodings which can be solved more quickly in the SAT solver.

## 2 Bounded Model Checking

### 2.1 Background

A model checking problem is a pair  $\langle M, f \rangle$  of a model and a temporal logic specification.

A model  $M$  is defined as a Kripke structure  $\langle S, R, L, I \rangle$  where  $S$  is a set of states;  $R : S \rightarrow S$  is the transition relation;  $L : S \rightarrow \mathcal{P}(AP)$  is the labelling function, marking each state with the set of atomic propositions ( $AP$ ) that hold in that state; and  $I$  is the set of initial states, which may be equal to  $S$ . A path  $\pi \in M$  is a sequence of states  $s_0, s_1, \dots \in M$  such that  $\forall i. (s_i, s_{i+1}) \in R$ . We write  $\pi(i)$  to refer to the  $i$ th state along the path.

The *model checking problem for LTL* is to verify that for an LTL formula  $f$ , for all paths  $\pi_i \in M$  such that  $\pi_i(0) \in I$ ,  $(M, \pi_i) \models f$ .

### 2.2 Path Loops

We say the a path  $\pi$  is a *k-loop* if for all  $i \geq 0$ , the  $(k+i)$ th state in  $\pi$  is identical to the  $l+i$ th state for some  $l, 0 \leq l < k$ . If a path is known to be a loop, it is possible to verify the correctness of infinite time specifications such as *always* ( $\mathbf{G}$ ) by checking just the first  $k$  states in the path.

### 2.3 Boolean Satisfiability

*Boolean satisfiability* (SAT) is the problem of assigning Boolean values to variables in a propositional formula, in such a way as to make the formula evaluate to true (to *satisfy* the formula). For example, for the formula  $(a \vee \neg b) \wedge (b \vee \neg c) \wedge (\neg c \vee \neg a)$  can be satisfied by *e.g.* the assignment  $a = 1, b = 1, c = 0$ .

SAT solvers derived from the Davis-Putnam algorithm [5] require input in clause form (CNF): a conjunction of *clauses*, each of which is a disjunction of literals.

A number of high performance SAT solvers are available, making SAT a convenient ‘black box’ back end for a number of different problems.

### 2.4 The Bounded Model Checking Encoding

The bounded model checking encoding represents  $k$  states along a bounded path  $\pi_{\text{bmc}}$  together with a conjunction of constraints requiring  $\pi_{\text{bmc}}$  to be a *valid path in M* and be a *counterexample of f*. The ‘valid path’ constraint is a propositional encoding of the transition relation. We can see from the bounded semantics of LTL (Figure 1) that there are two ways of violating each operator in the specification, depending on whether  $\pi_{\text{bmc}}$  is a  $k$ -loop; the ‘counterexample’ constraint is therefore a disjunction of the ways in which the specification may be violated.

We write the bounded model checking encoding of a problem with bound  $k$ , model  $M$  and specification  $f$  as

$$\llbracket M, \neg f \rrbracket_k$$

$$\begin{aligned}
(M, \pi) \models_k^i a &\Leftrightarrow a \in L(\pi(i)) \quad \text{for atomic } a \\
(M, \pi) \models_k^i \neg f_1 &\Leftrightarrow (M, \pi) \not\models_k^i f_1 \\
(M, \pi) \models_k^i f_1 \wedge f_2 &\Leftrightarrow (M, \pi) \models_k^i f_1 \text{ and } (M, \pi) \models_k^i f_2 \\
(M, \pi) \models_k^i f_1 \vee f_2 &\Leftrightarrow (M, \pi) \models_k^i f_1 \text{ or } (M, \pi) \models_k^i f_2 \\
(M, \pi) \models_k^i \mathbf{X} f_1 &\Leftrightarrow \begin{cases} (M, \pi) \models_k^{i+1} f_1 & \text{if } \pi \text{ is a } k\text{-loop} \\ (M, \pi) \models_k^{i+1} f_1 \wedge i < k & \text{otherwise} \end{cases} \\
(M, \pi) \models_k^i \mathbf{F} f_1 &\Leftrightarrow \begin{cases} \exists j, i \leq j. (M, \pi) \models_k^j f_1 & \text{if } \pi \text{ is a } k\text{-loop} \\ \exists j, i \leq j \leq k. (M, \pi) \models_k^j f_1 & \text{otherwise} \end{cases} \\
(M, \pi) \models_k^i \mathbf{G} f_1 &\Leftrightarrow \begin{cases} \forall j, i \leq j. (M, \pi) \models_k^j f_1 & \text{if } \pi \text{ is a } k\text{-loop} \\ \perp & \text{otherwise} \end{cases} \\
(M, \pi) \models_k^i [f_1 \mathbf{U} f_2] &\Leftrightarrow \begin{cases} \exists j, i \leq j. (M, \pi) \models_k^j f_2 \\ \quad \wedge \forall n, i \leq n < j. (M, \pi) \models_k^n f_1 & \text{if } \pi \text{ is a } k\text{-loop} \\ \exists j, i \leq j \leq k. (M, \pi) \models_k^j f_2 \\ \quad \wedge \forall n, i \leq n < j. (M, \pi) \models_k^n f_1 & \text{otherwise} \end{cases} \\
(M, \pi) \models_k^i [f_1 \mathbf{R} f_2] &\Leftrightarrow \begin{cases} \exists j, i \leq j. (M, \pi) \models_k^j f_1 \\ \quad \wedge \forall n, i \leq n \leq j. (M, \pi) \models_k^j f_2 & \text{if } \pi \text{ is a } k\text{-loop} \\ \exists j, i \leq j \leq k. (M, \pi) \models_k^j f_1 \\ \quad \wedge \forall n, i \leq n \leq j. (M, \pi) \models_k^j f_2 & \text{otherwise} \end{cases}
\end{aligned}$$

**Fig. 1.** The Bounded Semantics of LTL

**Table 1.** The BMC Encoding for LTL

$f$	$\llbracket f \rrbracket_k^0$	$\llbracket f \rrbracket_k^i$
$\mathbf{G} f_1$	$\perp$	$\bigwedge_{j=\min(i,l)}^k \llbracket f_1 \rrbracket_k^j$
$\mathbf{F} f_1$	$\bigvee_{j=i}^k \llbracket f_1 \rrbracket_k^j$	$\bigvee_{j=\min(i,l)}^k \llbracket f_1 \rrbracket_k^j$
$\mathbf{X} f_1$	$i < k \wedge \llbracket f_1 \rrbracket_k^{i+1}$	$(i < k \wedge \llbracket f_1 \rrbracket_k^{i+1}) \vee (i = k \wedge \llbracket f_1 \rrbracket_k^i)$
$f_1 \mathbf{U} f_2$	$\bigvee_{j=i}^k (\llbracket f_2 \rrbracket_k^j \wedge \bigwedge_{n=i}^{j-1} \llbracket f_1 \rrbracket_k^n)$	$\bigvee_{j=i}^k (\llbracket f_2 \rrbracket_k^j \wedge \bigwedge_{n=i}^{j-1} \llbracket f_1 \rrbracket_k^n)$ $\vee \bigvee_{j=l}^{i-1} (\llbracket f_2 \rrbracket_k^j \wedge \bigwedge_{n=i}^k \llbracket f_1 \rrbracket_k^n \wedge \bigwedge_{n=l}^{j-1} \llbracket f_1 \rrbracket_k^n)$
$f_1 \mathbf{R} f_2$	$\bigvee_{j=i}^k (\llbracket f_1 \rrbracket_k^j \wedge \bigwedge_{n=i}^j \llbracket f_2 \rrbracket_k^n)$	$\bigwedge_{j=\min(i,l)}^k \llbracket f_2 \rrbracket_k^j \vee \bigvee_{j=i}^k (\llbracket f_1 \rrbracket_k^j \wedge \bigwedge_{n=i}^j \llbracket f_2 \rrbracket_k^n)$ $\vee \bigvee_{j=l}^{i-1} (\llbracket f_1 \rrbracket_k^j \wedge \bigwedge_{n=i}^k \llbracket f_2 \rrbracket_k^n \wedge \bigwedge_{n=l}^j \llbracket f_2 \rrbracket_k^n)$

Given the functions  $\llbracket L_k(\pi) \rrbracket$  which holds when  $\pi$  is a  $k$ -loop with  $\pi(k) = \pi(l)$  and  $L_l(\pi) = \bigvee_{l=0}^k \llbracket L_k \rrbracket$  which holds when  $\pi$  is any  $k$ -loop, the general translation is defined as<sup>3</sup>:

$$\llbracket M, f \rrbracket_k := \llbracket M \rrbracket_k \wedge \left( \neg \llbracket L_k(\pi) \rrbracket \wedge \llbracket f \rrbracket_k^0 \vee \bigvee_{l=0}^k (\llbracket L_k(\pi) \rrbracket \wedge \llbracket f \rrbracket_k^l) \right) \quad (1)$$

where  $\llbracket M \rrbracket_k$  denotes the encoding of the transition relation of  $M$  as a constraint on  $\pi$  with bound  $k$ ;  $\llbracket f \rrbracket_i^k$  and  $\llbracket f \rrbracket_i^l$  denote the encoding of the LTL formula  $f$  evaluated along path  $\pi$  at time  $i$ , where  $\pi$  is a non-looping path and a  $k$ -loop to  $l$  respectively. These encodings are given in Table 1. Biere et al. show the correctness of some of these encodings in [2]; we will not repeat their proofs here.

Theorem 1 in Biere et al. [2] states that bounded model checking of this form is complete provided that the bound  $k$  is sufficiently large.

### 3 Exploiting Fixpoints in BMC

The approach that we have taken to making a fixpoint-based encoding for BMC is based on a clause-style normal form for temporal logic. After converting the specification to this form, we can redefine the encoding to specifically take advantage of the properties of the normal form.

#### 3.1 The Separated Normal Form

Gabbay's Separation Theorem [11] states that arbitrary temporal formulæ may be written in the form  $\mathbf{G} (\bigwedge_i (P_i \Rightarrow F_i))$  where  $P_i$  are (strict) past time formulæ and  $F_i$  are (non-strict) future time formulæ.

Fisher [9] defines a normal form for temporal logic based on the Separation Theorem and gave a series of transformations for reaching it. The general form of SNF is the same as the separation theorem; the implications  $P_i \Rightarrow F_i$  are referred

<sup>3</sup> This comes from Definition 15 in [2]

to as *rules*. Since neither LTL nor CTL have explicit past-time operators, Bolotov and Fisher [3] define the **start** operator which holds only at the beginning of time.

$$(M, \pi) \models_k^i \mathbf{start} \Leftrightarrow \pi(i) \in I$$

The possible rules are thus

$$\begin{array}{ll} \mathbf{start} \Rightarrow \bigvee_j l_j & \text{An } \textit{initial} \text{ rule} \\ \bigwedge_i l_i \Rightarrow \mathbf{X} \bigvee_j l_j & \text{A } \textit{global X-rule} \\ \bigwedge_i l_i \Rightarrow \mathbf{F} \bigvee_j l_j & \text{A } \textit{global F-rule} \end{array}$$

where  $l_i$  and  $l_j$  are literals.

The transformation functions  $T(\Psi)$  recursively convert a set of rules which do not conform to the normal form into a set of rules which do. To convert any temporal logic formula  $f$  to SNF, it is sufficient to apply the transformation rules to the singleton set  $\{\mathbf{start} \Rightarrow f\}$ . For brevity, we do not list the full set of transformations here; in general they are trivially adapted from those in [3], or from standard propositional logic.

$$\begin{aligned} T_G(\{P \Rightarrow \mathbf{G} f\} \cup \Psi) &= \left\{ \begin{array}{l} P \Rightarrow f \wedge x \\ x \Rightarrow \mathbf{X}(f \wedge x) \end{array} \right\} \cup T_G(\Psi) \\ T_U(\{P \Rightarrow f \mathbf{U} g\} \cup \Psi) &= \left\{ \begin{array}{l} P \Rightarrow g \vee (f \wedge x) \\ x \Rightarrow \mathbf{X}(g \vee (f \wedge x)) \\ P \Rightarrow \mathbf{F} g \end{array} \right\} \cup T_U(\Psi) \\ T_{\text{ren1}}(\{P \Rightarrow \mathbf{G} f(\mathbf{F} g)\} \cup \Psi) &= \left\{ \begin{array}{l} P \Rightarrow \mathbf{G} f(x) \\ x \Rightarrow \mathbf{F} g \end{array} \right\} \cup T_{\text{ren1}}(\Psi) \end{aligned}$$

In each of the above transformations, a new variable  $x$  is introduced: the conversion to SNF introduces one variable for each removed operator (in the first two transformations above) in addition to the renaming variables used to flatten the structure of the formula (in the last transformation above).

The transformations to rules are based on the fixpoint characterisations of the LTL operators. All LTL operators can be represented as the fixpoint of a recursive function [8]; the transformations encode the corresponding function as a rule which is required to hold in all states. Only those operators characterised by greatest fixpoints are converted (*always* (**G**) and *weak until* (**W**); *until* (**U**) is first converted to *weak until* and *sometime* for its transformation) which means that the *sometime* operator remains unchanged.

By Tarski's fixpoint theorem [18] we know that a finite number of iterations of a rule is sufficient to find its fixpoint. Thus the instance of the introduced variable at time  $i$  holds iff the original operator held at time  $i$ . For a formal proof of the correctness of the transformations, see [10].

### 3.2 Bounded SNF

Although the fixpoint characterisations are given for *unbounded* temporal logic, they are preserved for most of bounded LTL since we have bounded semantics for *next-state* ( $\mathbf{X}$ ). We note that the characterisation of *always* is valid if and only if the path is a  $k$ -loop; we encapsulate this constraint in the new operator *next-loop-state* ( $\mathbf{X}_1$ ) with semantics

$$(M, \pi) \models_k^i \mathbf{X}_1 f_1 \Leftrightarrow \begin{cases} (M, \pi(i+1)) \models_k f_1 & \text{if } \pi \text{ is a } k\text{-loop} \\ \perp & \text{otherwise} \end{cases}$$

and modify the transformation accordingly.

The bounded semantics of *always* also fails to capture the concept of rules holding in *all reachable states*. We give the semantics for a modified operator *bounded always* ( $\mathbf{G}_k$ ) for bounded LTL without the restriction to paths with loops.

$$(M, \pi) \models_k^i \mathbf{G}_k f_1 \Leftrightarrow \begin{cases} \forall j, i \leq j. (M, \pi(j)) \models_k f_1 & \text{if } \pi \text{ is a } k\text{-loop} \\ \forall j, i \leq j < k. (M, \pi(j)) \models_k f_1 & \text{otherwise} \end{cases}$$

The correctness of the transformations rely on a sufficient number of instances of the rules occurring. In BMC, this means that the transformations based on fixpoints are correct only when the bound is sufficiently large. It is easy to see, by appealing to the semantics, that the failure mode with an insufficiently large bound is the same as that for the original encoding: no counterexample is found.

Introducing this operator allows us to restate the general form as

$$\mathbf{G}_k \left( \bigwedge_i (P_i \Rightarrow F_i) \right)$$

The rules  $P_i \Rightarrow F_i$  are now of the following form:

$$\begin{array}{lll} \mathbf{start} \Rightarrow \bigvee_j l_j & \text{An } \textit{initial} \text{ rule} & \bigwedge_i l_i \Rightarrow \mathbf{X}_1 \bigvee_j l_j \quad \text{A } \textit{global } \mathbf{X}_1\text{-rule} \\ \bigwedge_i l_i \Rightarrow \mathbf{X} \bigvee_j l_j & \text{A } \textit{global } \mathbf{X}\text{-rule} & \bigwedge_i l_i \Rightarrow \mathbf{F} \bigvee_j l_j \quad \text{A } \textit{global } \mathbf{F}\text{-rule} \end{array}$$

with the transformation for the *always* operator being amended to

$$T_G(\{P \Rightarrow \mathbf{G} f\} \cup \Psi) = \left\{ \begin{array}{l} P \Rightarrow f \wedge x \\ x \Rightarrow \mathbf{X}_1(f \wedge x) \end{array} \right\} \cup T_G(\Psi)$$

The correctness of bounded SNF is covered in [16].

**Table 2.** The BMC Encoding for SNF-LTL

$f$	$\llbracket f \rrbracket_k^0$	$i\llbracket f \rrbracket_k^0$
$\mathbf{start} \Rightarrow f_1$	$\llbracket f_1 \rrbracket_k^0$	$i\llbracket f_1 \rrbracket_k^0$
$\mathbf{G}_k(f_1 \Rightarrow \mathbf{X}_1 f_2)$	$\perp$	$\bigwedge_{n=0}^k (i\llbracket f_1 \rrbracket_k^n \Rightarrow i\llbracket f_2 \rrbracket_k^{n+1})$
$\mathbf{G}_k(f_1 \Rightarrow \mathbf{X} f_2)$	$\bigwedge_{n=0}^{k-1} (\llbracket f_1 \rrbracket_k^n \Rightarrow \llbracket f_2 \rrbracket_k^{n+1})$	$\bigwedge_{n=0}^k (i\llbracket f_1 \rrbracket_k^n \Rightarrow i\llbracket f_2 \rrbracket_k^{n+1})$
$\mathbf{G}_k(f_1 \Rightarrow \mathbf{F} f_2)$	$\bigwedge_{n=0}^k (\llbracket f_1 \rrbracket_k^n \Rightarrow \bigvee_{m=n}^k \llbracket f_2 \rrbracket_k^m)$	$\bigwedge_{n=0}^k (i\llbracket f_1 \rrbracket_k^n \Rightarrow \bigvee_{m=\min(n,l)}^k i\llbracket f_2 \rrbracket_k^m)$

### 3.3 Encoding Bounded SNF

The distributivity of *bounded always* follows directly from its semantics; because of the unusual semantics of **start**, this means that any LTL formula may be represented as a conjunction of instances of the following ‘universal’ rules:

$$\begin{array}{ll}
 \mathbf{start} \Rightarrow \bigvee_j l_j & \mathbf{G}_k \left( \bigwedge_i l_i \Rightarrow \mathbf{X}_1 \bigvee_j l_j \right) \\
 \mathbf{G}_k \left( \bigwedge_i l_i \Rightarrow \mathbf{X} \bigvee_j l_j \right) & \mathbf{G}_k \left( \bigwedge_i l_i \Rightarrow \mathbf{F} \bigvee_j l_j \right)
 \end{array}$$

Although it is simple to encode these rules using the standard BMC encodings in Table 1, we can take advantage of the limited nesting depth characteristic of these normal forms to define a more efficient encoding, in the same way as for the depth 1 case in [4] and [17]. We give the more efficient encodings in Table 2. Note that although we make use of the BMC encodings, they are only used for purely propositional formulæ. No further proof of these encodings is required: they are trivial simplifications of those proved in [2].

For propositional  $f$ ,  $\llbracket f \rrbracket_k^i \equiv i\llbracket f \rrbracket_k^i$ , so we can deduce from Table 2 that this relationship also holds for many cases where  $f$  is a rule. Under these circumstances, we can factorise the encodings for  $f$  out of the disjunctions in Equation 1 either explicitly during the encoding or by processing the resulting propositional formula. Often the checks for the looping nature of  $\pi$  will cancel each other out entirely, further simplifying the encoding. While this type of optimisation can be made with the standard BMC encoding, it only occurs where operators are not nested; the renaming effect of SNF simplifies this optimisation and makes it more widely applicable.

### 3.4 The Fixpoint Normal Form

We noted in Section 3.1 that SNF converts only the greatest fixpoint operators, leaving rules containing the *sometime* operator; we see from Table 2 that these rules are the pathological case for this encoding. Converting the *sometime* operator in the same way requires care.

A transformation based directly on the fixpoint characterisation would be

$$T_F(\{P \Rightarrow \mathbf{F} f\} \cup \Psi) = \left\{ \begin{array}{l} P \Rightarrow f \vee x \\ x \Rightarrow \mathbf{X}(f \vee x) \end{array} \right\} \cup T_F(\Psi)$$

The problem stems from the disjunction in the second rule. Since we are trying to show satisfiability, it is simple to satisfy each occurrence of the rule by setting the right hand disjunct to true for all time: the rule can always be satisfied. Since we are interested only in the bounded semantics of the operator, it is possible to break this chain at the bound by introducing an extra operator:

$$(M, \pi) \models_k^i \mathbf{bound} \Leftrightarrow i \geq k$$

The transformation is now

$$T_F(\{P \Rightarrow \mathbf{F} f\} \cup \Psi) = \left\{ \begin{array}{l} P \Rightarrow f \vee x \\ x \Rightarrow \mathbf{X}(f \vee x) \\ \mathbf{bound} \Rightarrow f \vee \neg x \end{array} \right\} \cup \Psi$$

### 3.5 Correctness of the Fixpoint Normal Form Transformation

We take the outline of the proof from [10]. For a transformation  $T$  to preserve the semantics of an arbitrary formula  $f$ , we require that

for all models  $M$  and for all LTL formulæ  $f$ ,  $(M, \pi) \models_k f$  iff there exists an  $M'$  such that  $M \sim^x M'$  and  $(M, \pi) \models_k \tau(f)$

where  $x$  is a new propositional variable introduced, and  $M \sim^x M'$  if and only if  $M$  differs from  $M'$  in at most the valuation given  $x$ . We express this in temporal logic with quantification over propositions (QPTL)<sup>4</sup> as  $\vdash_{\text{QPTL}} f \Leftrightarrow \exists x.T(f)$ . The proof is given for the case that the rule set is a singleton set, since for all transformations,  $T$  is independent of  $\Psi$ . The proofs may easily be extended to non-empty  $\Psi$ .

**Lemma 1.** *For sufficiently large  $k$ ,  $(M, \pi) \models_k \mathbf{F} f_1$  if and only if  $(M', \pi) \models_k (x \vee f_1)$  and  $(M', \pi) \models_k \mathbf{G}_k(x \Leftrightarrow \mathbf{X}(x \vee f_1))$  where  $M \sim^x M'$ .*

*Proof.* Consider the fixpoint expression  $\tau(Z) = f_1 \vee \mathbf{X}Z$ . We introduce the variable  $x$  such that for all  $n$ ,

$$(M', \pi) \models_k^n x \Leftrightarrow (M', \pi) \models_k^n \mathbf{X} \tau^{k-n}(\text{true})$$

By substituting the definition of  $x$  and by one substitution of the definition of  $\tau$ , we have  $(M', \pi) \models_k^n x \Leftrightarrow (M', \pi) \models_k^n \mathbf{X}(f_1 \vee x)$  and by reference to the semantics,  $(M', \pi) \models_k \mathbf{G}_k(x \Leftrightarrow \mathbf{X}(x \vee f_1))$ .

From the least fixpoint characterisation[8],  $(M', \pi) \models_k x \Leftrightarrow \mathbf{F} f_1$ , and by unrolling  $\tau$  by one step and substituting the definition of  $x$ , we get  $(M', \pi) \models_k f_1 \vee x$ .

<sup>4</sup> See [19] for full details; briefly,  $(M, i) \models \exists p.A$  iff there exists an  $M'$  such that  $(M', i) \models A$ , and  $M'$  and  $M$  differ at most in the valuation given to  $p$ .

**Theorem 1.** For any rule  $A$ ,  $\vdash_{\text{QPTL}} A \Leftrightarrow \exists x.T_F(A)$

*Proof.* Proving each direction independently:

- $\vdash_{\text{QPTL}} A \Rightarrow \exists x.T_F(A)$   
Substituting Lemma 1,

$$\begin{aligned} & \mathbf{G}_k(P \Rightarrow \mathbf{F} B) \\ & \Rightarrow \exists x.\mathbf{G}_k(x \Leftrightarrow \mathbf{X}(x \vee B)) \wedge \mathbf{G}_k(\mathbf{bound} \Rightarrow \neg x) \wedge \mathbf{G}_k(P \Rightarrow (x \vee B)) \\ & \Rightarrow \exists x.\mathbf{G}_k((x \Leftrightarrow \mathbf{X}(x \vee B)) \wedge \mathbf{bound} \Rightarrow \neg x \wedge (P \Rightarrow (x \vee B))) \end{aligned}$$

which implies the set of rules  $\{x \Rightarrow \mathbf{X}(x \vee B), \mathbf{bound} \Rightarrow \neg x, P \Rightarrow x \vee B\}$ .

- $\vdash_{\text{QPTL}} \exists x.T_F(f) \Rightarrow f$   
Starting with the transformed set of rules  $\{x \Rightarrow \mathbf{X}(x \vee B), \mathbf{bound} \Rightarrow \neg x, P \Rightarrow x \vee B\}$ , and exploiting the corollary of Lemma 1,  $(M', s_i) \models_k (x \vee f_1) \Leftrightarrow (M', s_i) \models_k \mathbf{F} f_1$  iff  $(M', s_i) \models \mathbf{G}_k(\mathbf{bound} \Rightarrow \neg x)$

$$\begin{aligned} & \mathbf{G}_k((x \Leftrightarrow \mathbf{X}(x \vee B)) \wedge \mathbf{bound} \Rightarrow \neg x \wedge (P \Rightarrow (x \vee B))) \\ & \Leftrightarrow \mathbf{G}_k(x \Leftrightarrow \mathbf{X}(x \vee B)) \wedge \mathbf{G}_k(\mathbf{bound} \Rightarrow \neg x) \wedge \mathbf{G}_k(P \Rightarrow (x \vee B)) \\ & \Leftrightarrow \mathbf{G}_k(x \Leftrightarrow \mathbf{X} \mathbf{F} B) \wedge \mathbf{G}_k(\mathbf{bound} \Rightarrow \neg x) \wedge \mathbf{G}_k(P \Rightarrow (x \vee B)) \\ & \Rightarrow \mathbf{G}_k((x \Rightarrow \mathbf{X} \mathbf{F} B) \wedge (P \Rightarrow (x \vee B))) \\ & \Rightarrow \mathbf{G}_k(P \Rightarrow ((\mathbf{X} \mathbf{F} B) \vee B)) \\ & \Rightarrow \mathbf{G}_k(P \Rightarrow \mathbf{F} B) \end{aligned}$$

That is, the singleton rule set  $\{P \Rightarrow \mathbf{F} B\}$ .

## 4 Comparisons

We compare the encodings on an example specification  $\mathbf{G} \mathbf{F} f$ . This is a reachability specification, with many applications. Before encoding, the specification is negated to

$$\mathbf{F} \mathbf{G} \neg f \tag{2}$$

We consider only the loop encoding, as the non-loop encoding is  $\perp$  for all methods due to the semantics of *always*.

The original, recursive encoding decomposes in two steps. In the loop case,

$$\begin{aligned} {}_l[\mathbf{F} \mathbf{G} \neg f, \pi]_k^0 &= \bigvee_{i=0}^k {}_l[\mathbf{G} \neg f, \pi]_k^i \\ &= \bigvee_{i=0}^k \bigwedge_{j=\min(i,l)}^k f(j) \end{aligned}$$

This is a disjunction of conjunctions: the pathological case for conversion to clause form. It is possible to define a more efficient encoding using renamed

subformulæ [4], but this approach is difficult to generalise. The size of the formula is  $O(k^2)$ , hence the cost to build it before CNF conversion is quadratic.

The conversion to SNF yields the following rules<sup>5</sup>

$$\begin{aligned} \mathbf{start} &\Rightarrow \mathbf{F} x_1 \\ x_1 &\Rightarrow \neg f \wedge x_2 \\ x_2 &\Rightarrow \mathbf{X}_1(\neg f \wedge x_2) \end{aligned}$$

which encode to the three conjuncts

$$\begin{aligned} &\bigvee_{i=0}^k x_1(i) \quad \wedge \\ &\bigwedge_{i=0}^k (x_1(i) \Rightarrow \neg f(i) \wedge x_2(i)) \quad \wedge \\ &\bigwedge_{i=0}^k (x_2(i) \Rightarrow \neg f(i+1) \wedge x_2(i+1)) \end{aligned}$$

We have two introduced variables: the first establishes a renaming of the  $\mathbf{G} \neg f$  subformula, and the second renames each successive step of this subformula. This means that steps are shared between references from the  $\mathbf{F}$  operator, leading to a simplification of the problem which is easier to solve as well as being smaller. The added complexity of the introduced variables is balanced by the ability to reuse subformulæ many times.

The encoding corresponds to an ideal renaming of the formula above, but the conversion is performed in linear time, and results in a formula of size  $O(k)$ . Furthermore, we can show in advance that the encoding of each rule used here is invariant with respect to  $l$ , which means that the subformulæ can be factorised out of the disjunction of loops seen in Equation 1.

Finally, we examine the fixpoint normal form conversion. The set of rules corresponding to the specification is

$$\begin{aligned} \mathbf{start} &\Rightarrow x_0 \vee x_1 \\ x_0 &\Rightarrow \mathbf{X}(x_0 \vee x_1) \\ \mathbf{bound} &\Rightarrow x_1 \vee \neg x_0 \\ x_1 &\Rightarrow \neg f \wedge x_2 \\ x_2 &\Rightarrow \mathbf{X}_1(\neg f \wedge x_2) \end{aligned}$$

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<sup>5</sup> Further reduction of the second and third rules is necessary for correct SNF; we disregard this as it makes no difference to the final encoding

which encode to the conjuncts

$$\begin{aligned}
& x_0(0) \vee x_1(0) \quad \wedge \\
& \bigwedge_{i=0}^k (x_0(i) \Rightarrow x_0(i+1) \vee x_1(i+1)) \quad \wedge \\
& x_1(k) \vee \neg x_0(k) \quad \wedge \\
& \bigwedge_{i=0}^k (x_1(i) \Rightarrow \neg f(i) \wedge x_2(i)) \quad \wedge \\
& \bigwedge_{i=0}^k (x_2(i) \Rightarrow \neg f(i+1) \wedge x_2(i+1))
\end{aligned}$$

The main difference between the SNF encoding and the fixpoint normal form encoding is the omission of the long disjunction in the first conjunct which would be encoded as a single long clause. This is replaced by an array of conjunctions which rename each step in much the same way as for the  $\mathbf{G}$  operator. Although in this case the advantage is dependent on the SAT checker, it is clear that where the  $\mathbf{F}$  operator is nested, similar advantages would be seen as for SNF with the  $\mathbf{G}$  operator.

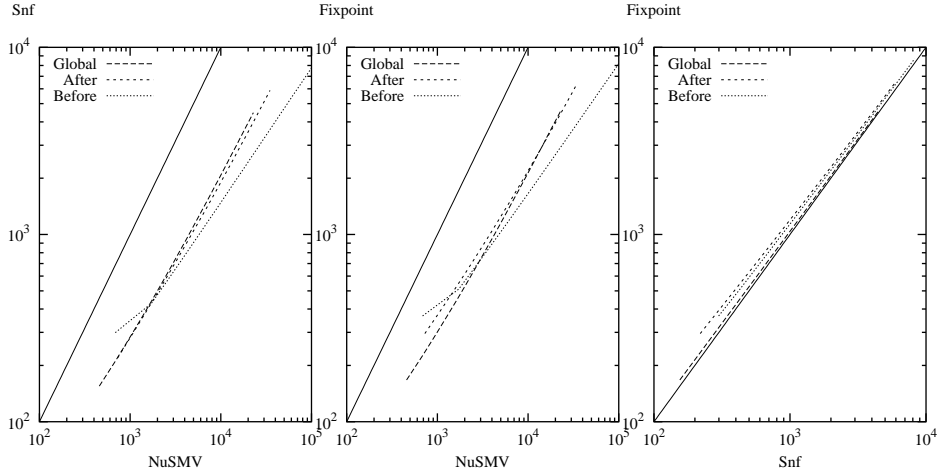
## 5 Results

We compare the SNF and Fixpoint encodings with the encoding used in NuSMV version 2.0.2; this version of NuSMV includes several of the optimisations discussed in [4]. For consistency, we have implemented the SNF and Fixpoint encodings as options in NuSMV. All of the experiments have been done using the SAT solver zChaff [15] on a 700MHz AMD Athlon with 256Mb main memory, running Linux.

### 5.1 Scalability

We observe the difference in the behaviour of the encodings with increasing problem size by choosing a simple problem that is easy to scale. The benchmark circuits have been kept deliberately simple as it is the encoding of the specification not the model that differentiates the encodings.

A shift register is a storage device of length  $n$  which, at each time step, moves the values stored in each of its elements to the next element, reading in a new value to fill the now empty first element. That is, storage elements  $x_0 \dots x_{n-1}$  and input  $in$  are transformed such that  $\forall i, 0 < i < n \cdot (x_i \leftarrow x_{i-1})$  and  $x_0 \leftarrow in$ . The specification that the shift register must fulfil will depend on its application; we explore a number of response patterns taken from [6]. The specifications depend on the number of elements in the shift register, referring to points at the end and middle of the register. For example, in the case of a three element register:



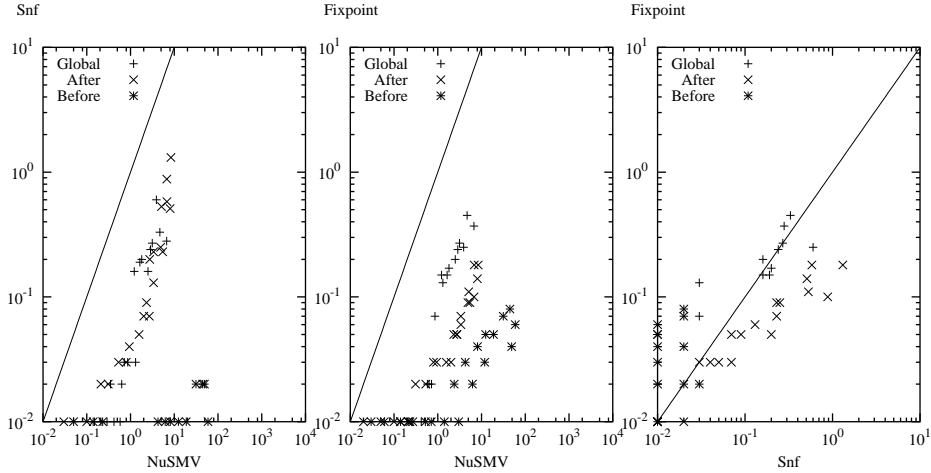
**Fig. 2.** Number of clauses generated by a shift register model

- Global response (depth 2) —  $x_2$  goes high in response to  $in$ :  $\mathbf{G}(in \Rightarrow \mathbf{F} x_2)$
- After response (depth 3) —  $x_2$  goes high in response to  $in$ , after  $x_1$  has gone high:  $\mathbf{G}(x_1 \Rightarrow \mathbf{G}(in \Rightarrow \mathbf{F} x_2))$
- Before response (depth 3) —  $x_1$  goes high in response to  $in$ , before  $x_2$  has gone high (this property is only true if all the registers are zero, so we test for  $empty \equiv \neg x_0 \wedge \neg x_1 \wedge \neg x_2$  too):  $[(in \wedge empty) \Rightarrow [\neg x_2 \mathbf{U}(x_1 \wedge \neg x_2)]] \mathbf{U}(x_2 \vee \mathbf{G} x_2)$

**Number of Clauses.** We see in Figure 2 that the number of clauses produced by both SNF and Fixpoint grows, in general, less quickly than the number produced by NuSMV, as the length of the register increases. The differing gradients follow the behaviour predicted by the differing depths of the specifications: the slopes become shallower with increasing depth indicating an exponential improvement in the number of clauses.

The advantage of the Fixpoint encoding over SNF is dependent upon the number of occurrences of the *always* operator in the specification, since this is the only difference between the encodings. We see the greatest advantage for Fixpoint in the after response and before response specifications, with two occurrences of the *always* operator; the first operator in the after response specification has a smaller encoding than the second as one of the corresponding rules is an initial rule.

We can conclude that, as far as the number of clauses is concerned, the Fixpoint encoding outperforms SNF and NuSMV in the way that is expected: size and rate of size increase decreasing with the nesting depth and the occurrence of least fixpoint operators.



**Fig. 3.** Time taken by zChaff for a shift register model

**zChaff timings.** Counting the number of clauses is far from being an effective method of determining the efficiency of an encoding. We also look at one of the current state-of-the-art SAT solvers, zChaff [15].

The behaviour is far less clear than for the number of clauses; zChaff is a complex system. Broadly, the SNF and Fixpoint encodings always result in a shorter runtime than the NuSMV encoding; the Fixpoint encoding outperforms the SNF encoding only for the after response specification (for the global response specification, the trend is towards an improvement for larger problems).

We see a clear exponential improvement for certain specifications: the timings for Before with SNF and Fixpoint grow exponentially slower than NuSMV; the global response specification shows the same trend less dramatically. We only see an exponential improvement for the after response specification with the Fixpoint encoding: with the SNF encoding, the trend appears to be towards NuSMV being faster.

## 5.2 Distributed Mutual Exclusion

The distributed mutual exclusion circuit from [13] forms a good basis for comparing the performance of different encodings as it meaningfully implements several specifications. We look at three here, applied to a DME of four elements:

- Accessibility: if an element wishes to enter the critical region, it eventually will. We check the accessibility of the first two elements. This specification is correct, so as in [2], we check at a chosen bound to illustrate the timing differences.

$$\mathbf{G}(\text{request}(0) \rightarrow \mathbf{F} \text{enter}(0)) \wedge \mathbf{G}(\text{request}(1) \rightarrow \mathbf{F} \text{enter}(1))$$

- Precedence given token possession: the mutual exclusion property is enforced by a token passing mechanism; if an element of the DME holds the token,

**Table 3.** Timing results in zChaff for the distributed mutual exclusion circuit

Specification	Bound	NuSMV encoding	SNF encoding	Fixpoint encoding	SMV
Accessibility	30	2.65	0.33	0.36	13.13
Accessibility	40	20.93	4.84	4.33	13.13
Priority for 0	14	0.13	0.02	0.02	12.97
Priority for 1	54	14.93	0.44	0.76	15.00
Overtaking depth 1	40	85.73	2.15	1.11	13.96
Overtaking depth 2	40	*	4.92	5.15	14.14

then its requests to enter the critical region are given precedence. We check the converse: if the first element holds the token, the second does not have precedence and *vice versa*. Since the token begins at the first element, this is the quicker to prove, with a bound of 14. For the second element, a bound of 54 is required to find the counterexample.

$\mathbf{G}((request(0) \wedge request(1) \wedge token(0)) \rightarrow [\neg enter(0) \mathbf{U} enter(1)])$

- Bounded overtaking given token possession: if two elements wish to enter the critical region, then the higher priority may enter a given number of times before the other. We check bounded overtaking of one and two entrances. Both specifications are correct so as above we check at a bound of 40. These specifications are the most complex, including up to four nested *until* operators.

For one entrance:  $\mathbf{G}((request(0) \wedge request(1) \wedge token(0)) \rightarrow [(\neg enter(0) \wedge \neg enter(1)) \mathbf{U} (enter(0) \wedge \mathbf{X}(enter(0) \mathbf{U} [(\neg enter(0) \wedge \neg enter(1)) \mathbf{U} enter(1)])])])$

The results are summarised in Table 3 together with the timings for CMU SMV on CTL representations of the same problems<sup>6</sup>. For the bounded overtaking problems, we note that NuSMV took nearly 10 minutes to generate the formula in the first case, and after 25 minutes had not completed in the second case. In contrast, the time taken to perform the SNF and Fixpoint encodings were insignificant.

While both the SNF and Fixpoint encodings outperform the NuSMV encoding and SMV, we do not see a consistent advantage to either. The results for accessibility suggest that Fixpoint scales better with increasing bound, while the results for bounded overtaking suggest that SNF scales better with increasing specification depth.

### 5.3 Texas-97 Benchmarks

We examine a number of model checking benchmarks from the *Texas-97* benchmark suite [1]. These benchmarks have been converted from the Blif-mv representation to SMV format by a locally modified version of the VIS model checker [12]. We run these benchmarks at fixed bounds and report the time spent by zChaff.

<sup>6</sup> We note that for SMV to terminate in a reasonable time on these problems, it must be started with the `-inc` switch. No similar knowledge of model checker behaviour is needed for BMC.

**Table 4.** Timing results in zChaff for the MSI cache coherence protocol

Processors	Specification	Bound	NuSMV	SNF	Fixpoint
2	Request A	10	4.40	1.73	1.53
2	Request A	20	19.40	5.82	9.97
2	Request B	10	2.65	3.63	2.69
2	Request B	20	49.78	8.63	16.42
3	Request A	10	13.00	3.03	2.50
3	Request A	20	39.22	8.2	5.79
3	Request B	10	4.60	6.66	5.93
3	Request B	20	54.94	62.11	40.25
3	Request C	10	4.58	6.64	5.91
3	Request C	20	44.8	50.27	37.65

**MSI Cache Coherence Protocol** This is an implementation of a Modified Shared Invalid cache coherence protocol between two or three processors. We examine two of the specifications of behaviour from the benchmark. The results are summarised in Table 4.

- Whenever processor A requests the bus, it gets the bus in the next clock cycle. Listed as “Request A” in the results.  $\mathbf{G}(bus\_reqA \rightarrow \mathbf{X} bus\_ackA)$
- Whenever processor B (or C) requests the bus, it gets the bus only when Processor A did not request the bus. Listed as “Request B” or “Request C” in the results.  $\mathbf{G}(bus\_reqB \rightarrow \mathbf{F} bus\_ackB)$

**Instruction Fetch Control Module** This is a model of the instruction fetch control module of the experimental TORCH microprocessor developed at Stanford University. Three models are examined; from the text accompanying the benchmark set:

- IFetchControl1: The original instruction module with several assumptions on the environmental signal.
- IFetchControl2: As IFetchControl1 except that the memory stall signal is always low.
- IFetchControl3.v: As IFetchControl1 except that the instruction cache line is assumed to be always valid.

We examine three specifications from the benchmark. The results are summarised in Table 5.

- The delayed version of a signal should, in the next state, have the signal’s previous value. Listed as “Delay” in the results.  $\mathbf{G}(IStall\_s1 \rightarrow \mathbf{X} IStall\_s2)$
- As above, for the Refetch state. Listed as “Refetch” in the results.  $\mathbf{G}((PresState\_s1 = REFETCH) \rightarrow \mathbf{X}((PrevState\_s2 = REFETCH)))$
- $WriteCache\_s2$  becomes one in some paths before  $WriteTag\_s2$  becomes one. Listed as “WriteCache” in the results.  $\neg[\neg WriteTag\_s2 \mathbf{U} (WriteCache\_s2 \wedge \neg WriteTag\_s2)]$

**Table 5.** Timing results in zChaff for the Instruction Fetch Control Module

Model	Specification	Bound	NuSMV	SNF	Fixpoint
IFetchControl1	Delay	10	0.94	0.45	0.44
IFetchControl2	Delay	10	0.99	0.40	0.40
IFetchControl3	Delay	10	1.29	0.39	0.50
IFetchControl1	Refetch	10	3.69	0.91	0.82
IFetchControl2	Refetch	10	3.30	0.89	0.81
IFetchControl3	Refetch	10	3.74	1.49	1.88
IFetchControl1	WriteCache	10	3.58	1.68	2.47
IFetchControl2	WriteCache	10	2.67	1.65	1.78
IFetchControl3	WriteCache	10	2.78	2.24	1.40

**Table 6.** Timing results in zChaff for the Pentium Pro Split-Transaction Bus

Opcode 0	Opcode 1	Specification	NuSMV	SNF	Fixpoint
Load2Store	Load2Store	IOQ	949.53	202.83	202.90
Load2Store	Store	IOQ	753.26	156.43	156.24
Load2Store	Load2Store	Live 1	923.12	176.64	169.97
Load2Store	Load	Live 1	745.94	131.61	145.88
Load2Store	Store	Live 1	1111.63	175.58	199.19
Load2Store	Load2Store	Live 2	919.61	167.23	160.38
Load2Store	Load	Live 2	883.51	134.52	155.23
Load2Store	Store	Live 2	738.74	128.96	143.04

**Pentium Pro Split-Transaction Bus** This is a model of the Modified Exclusive Shared Invalid cache coherence protocol used by the Intel Pentium Pro processor for SMP. We examine a number of different combinations of opcodes running on the processors, with the memory address of the transaction being nondeterministically 0 or 1.

We examine three specifications from the benchmark. The results are summarised in Table 6.

- Correctness of the bus transaction IOQ. Listed as “IOQ” in the results.  
 $\mathbf{G}(\neg((processor0.fifo = REQUEST) \wedge (processor1.fifo = REQUEST)))$
- Liveness of processor 0 (part 1). Listed as “Live 1” in the results.  
 $\mathbf{G}((processor0.stage = FETCH) \rightarrow \mathbf{F}(processor0.stage = EXECUTE))$
- Liveness of processor 0 (part 2). Listed as “Live 2” in the results.  
 $\mathbf{G}((processor0.stage = EXECUTE) \rightarrow \mathbf{F}(processor0.stage = FETCH))$

**Summary** While we can see that the SNF and Fixpoint encodings outperform the NuSMV encoding in many cases, the gains are typically less dramatic than were seen for the mutual exclusion circuit. The models are encoded in the same way regardless of the encoding used for the specification; these benchmarks are very large circuits with several thousand variables, so it is reasonable to suppose that the performance gains due to the new encodings are mitigated by the time taken to process the model.

The specifications used in these benchmarks are much simpler than those used to test the DME: typically of the form  $\mathbf{G}(a \rightarrow \mathbf{X} b)$  or  $\mathbf{G}(a \rightarrow \mathbf{F} b)$ . This suggests again that one advantage of the SNF and Fixpoint encodings are dependent on the nesting depth of the specification.

## 6 Conclusions

We have described two new encoding schemes for bounded model checking which build on the existing encodings and use the fixpoint characterisations of LTL. The first is a novel application of the Separated Normal Form, while the second extends SNF by the introduction of a transformation for the *eventually* operator. We have shown that these new encodings are correct, provided that the original bounded model checking encoding is correct.

We have demonstrated a reduction in the number of clauses generated by the problem which is exponential in the size of the problem instance, for both encodings, and also that the improvement in performance in the SAT checker can be exponential in the size of the problem instance, depending on the specification. We have demonstrated a clear performance advantage to these encodings over the NuSMV bounded model checking implementation in several real-world examples, and we have demonstrated the advantage that these encodings give BMC over conventional symbolic model checkers.

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